Formula sheet

Co-occurrence matrix $g(i,j) = \{$no. of pixel pairs with grey levels (z_i, z_j) satisfying predicate $Q\}$, $1 \leq i,j \leq L$

Convolution, 2-D discrete $(f \star h)(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, h(x-m, y-n)$, for $x = 0,1,2,\ldots,M-1, y = 0,1,2,\ldots,N-1$

Convolution Theorem, 2-D discrete $\mathcal{F}\{f \star h\}(u,v) = F(u,v)\, H(u,v)$

Distance measures Euclidean: $D_e(p,q) = \sqrt{(p_1-q_1)^2 + (p_2-q_2)^2}$, City-block: $D_4(p,q) = |p_1 - q_1| + |p_2 - q_2|$, Chessboard: $D_8(p,q) = \max(|p_1 - q_1|, |p_2 - q_2|)$

Entropy, source $H = -\sum_{j=1}^{J} P(a_j) \log P(a_j)$

Entropy, estimated for L-level image: $\tilde{H} = -\sum_{k=0}^{L-1} p_r(r_k) \log_2 p_r(r_k)$

Error, root-mean square $e_{\text{rms}} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\widehat{f}(x,y) - f(x,y)\right)^2\right]^{\frac{1}{2}}$

Exponentials $e^{ix} = \cos x + i \sin x; \quad \cos x = (e^{ix} + e^{-ix})/2; \quad \sin x = (e^{ix} - e^{-ix})/2i$

Filter, inverse $\widehat{\mathbf{f}} = \mathbf{f} + \mathbf{H}^{-1}\mathbf{n}$, $\widehat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$

Filter, parametric Wiener $\widehat{\mathbf{f}} = (\mathbf{H}^t\mathbf{H} + K\,\mathbf{I})^{-1}\, \mathbf{H}^t\mathbf{g}$, $\widehat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + K}\right] G(u,v)$

Fourier series of signal with period T: $f(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{i\frac{2\pi n}{T}t}$, with Fourier coefficients: $c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-i\frac{2\pi n}{T}t}\, dt, \quad n = 0, \pm1, \pm2, \ldots$

Fourier transform 1-D (continuous) $F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-i2\pi\mu t}\, dt$

Fourier transform 1-D, inverse (continuous) $f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{i2\pi\mu t}\, d\mu$

Fourier Transform, 2-D Discrete $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-i2\pi(u\,x/M + v\,y/N)}$ for $u = 0,1,2,\ldots,M-1, v = 0,1,2,\ldots,N-1$

Fourier Transform, 2-D Inverse Discrete $f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{i2\pi(u\,x/M + v\,y/N)}$ for $x = 0,1,2,\ldots,M-1, y = 0,1,\ldots,N-1$

Fourier spectrum Fourier transform of $f(x,y)$: $F(u,v) = R(u,v) + i\,I(u,v)$, Fourier spectrum: $|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$, phase angle: $\phi(u,v) = \arctan\left(\frac{I(u,v)}{R(u,v)}\right)$

Gaussian function mean μ, variance σ^2: $G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2}$

Gradient $\nabla f(x,y) = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$

Histogram $h(m) = \#\{(x,y) \in D : f(x,y) = m\}$. Cumulative histogram: $\mathsf{P}(\ell) = \sum_{m=0}^{\ell} h(m)$

Impulse, discrete $\delta(0) = 1, \delta(x) = 0$ for $x \in \mathbb{N}\backslash\{0\}$

Impulse, continuous $\delta(\infty) = 1, \delta(x) = 0$ for $x \neq 0$, with $\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0)$

Impulse train $s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$, with Fourier transform $S(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta(\mu - \frac{n}{\Delta T})$

Laplacian $\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

Laplacian-of-Gaussian $\nabla^2 G_\sigma(x,y) = -\frac{2}{\pi\sigma^4}\left(1 - \frac{r^2}{2\sigma^2}\right) e^{-r^2/2\sigma^2} \quad (r^2 = x^2 + y^2)$

Morphology

Dilation $\delta_A(X) = X \oplus A = \bigcup_{a\in A} X_a = \bigcup_{x\in X} A_x = \{h \in E : \check{A}_h \cap X \neq \emptyset\}$,
where $X_h = \{x + h : x \in X\}$, $h \in E$ and $\check{A} = \{-a : a \in A\}$

Erosion $\varepsilon_A(X) = X \ominus A = \bigcap_{a\in A} X_{-a} = \{h \in E : A_h \subseteq X\}$

Opening $\gamma_A(X) = X \circ A := (X \ominus A) \oplus A = \delta_A\varepsilon_A(X)$

Closing $\phi_A(X) = X \bullet A := (X \oplus A) \ominus A = \varepsilon_A\delta_A(X)$

Hit-or-miss transform $X \circledast (B_1, B_2) = (X \ominus B_1) \cap (X^c \ominus B_2)$

Thinning $X \otimes B = X \backslash (X \circledast B)$, **Thickening** $X \odot B = X \cup (X \circledast B)$

Morphological reconstruction Marker F, mask G, structuring element B:
$X_0 = F$, $X_k = (X_{k-1} \oplus B) \cap G, \quad k = 1, 2, 3, \ldots$

Morphological skeleton Image X, structuring element B: $SK(X) = \bigcup_{n=0}^{N} S_n(X)$,
$S_n(X) = X \underset{n}{\ominus} B \setminus (X \underset{n}{\ominus} B) \circ B$, $S_0(X) = X$, with N the largest integer such that $S_N(X) \neq \emptyset$

Grey value dilation $(f \oplus b)(x, y) = \max\limits_{(s,t)\in B} [f(x - s, y - t) + b(s, t)]$

Grey value erosion $(f \ominus b)(x, y) = \min\limits_{(s,t)\in B} [f(x + s, y + t) - b(s, t)]$

Grey value opening $f \circ b = (f \ominus b) \oplus b$

Grey value closing $f \bullet b = (f \oplus b) \ominus b$

Morphological gradient $g = (f \oplus b) - (f \ominus b)$

Top-hat filter $T_{\text{hat}} = f - (f \circ b)$, **Bottom-hat filter** $B_{\text{hat}} = (f \bullet b) - f$

Sampling of continuous function $f(t)$: $\tilde{f}(t) = f(t)\, s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta T)$.
Fourier transform of sampled function: $\tilde{F}(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F(\mu - \frac{n}{\Delta T})$

Sampling theorem Signal $f(t)$, bandwidth $\mu_{\max}$: If $\frac{1}{\Delta T} \geq 2\mu_{\max}$, $f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta T)\, \text{sinc}\left[\frac{t - n\Delta T}{n\Delta T}\right]$.

Sampling: downsampling by a factor of 2: $\downarrow_2 (a_0, a_1, a_2, \ldots, a_{2N-1}) = (a_0, a_2, a_4, \ldots, a_{2N-2})$

Sampling: upsampling by a factor of 2: $\uparrow_2 (a_0, a_1, a_2, \ldots, a_{N-1}) = (a_0, 0, a_1, 0, a_2, 0, \ldots, a_{N-1}, 0)$

Set, circularity ratio $R_c = \frac{4\pi A}{P^2}$ of set with area A, perimeter P

Set, diameter $\text{Diam}(B) = \max\limits_{i,j} [D(p_i, p_j)]$ with p_i, p_j on the boundary B and D a distance measure

Sinc function $\text{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$ when $x \neq 0$, and $\text{sinc}\,(0) = 1$

Spatial moments of an $M \times N$ image $f(x, y)$: $m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p\, y^q\, f(x, y), \quad p, q = 0, 1, 2, \ldots$

Statistical moments of distribution $p(i)$: $\mu_n = \sum_{i=0}^{L-1} (i - m)^n\, p(i)$, $m = \sum_{i=0}^{L-1} i\, p(i)$

Signal-to-noise ratio, mean-square $\text{SNR}_{\text{rms}} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\hat{f}(x,y) - f(x,y)\right)^2}$

Wavelet decomposition with low pass filter h_ϕ, band pass filter h_ψ. For $j = 1, \ldots, J$:
Approximation: $c_j = \mathbf{H} c_{j-1} = \downarrow_2 (h_\phi * c_{j-1})$; Detail: $d_j = \mathbf{G} c_{j-1} = \downarrow_2 (h_\psi * c_{j-1})$

Wavelet reconstruction with low pass filter $\tilde{h}_\phi$, band pass filter $\tilde{h}_\psi$. For $j = J, J - 1, \ldots, 1$:
$c_{j-1} = \tilde{h}_\phi * (\uparrow_2 c_j) + \tilde{h}_\psi * (\uparrow_2 d_j)$

Wavelet, Haar basis $h_\phi = \frac{1}{\sqrt{2}}(1, 1)$, $h_\psi = \frac{1}{\sqrt{2}}(1, -1)$, $\tilde{h}_\phi = \frac{1}{\sqrt{2}}(1, 1)$, $\tilde{h}_\psi = \frac{1}{\sqrt{2}}(1, -1)$